USN | | | | | | | | | | **13MCA12**

First Semester MCA Degree Examination, Dec.2016/Jan.2017
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Indicate how many rows are needed in the truth table for the compound proposition $(p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \rightarrow t\}$. Find the truth table of this proposition if p and r are true and q, s, t are false. **(05 Marks)**

b. Prove that, for any propositions p, q, r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is tautology. **(05 Marks)**

c. Write down the negation of the proposition : "If x is not a real number, then it is not a rational number and not an irrational number". **(05 Marks)**

d. Prove the result using laws of logic, $\neg[\{(p \vee q) \wedge r\} \rightarrow \neg q] \Leftrightarrow q \wedge r$. **(05 Marks)**

2 a. Let $p(x) : x^2 - 7x + 10 = 0$, $q(x) : x^2 - 2x - 3 = 0$, $r(x) : x < 0$. Determine the truth or falsity of the following statements when the universe contains only the integers 2 and 5. If a statement is false, provide a counter example.

i) $\forall x, p(x) \rightarrow \neg r(x)$
ii) $\forall x, q(x) \rightarrow r(x)$
iii) $\exists x, p(x) \rightarrow r(x)$. **(06 Marks)**

b. Test the validity of the argument. "If I study, I will not fail in the examination. If I do not watch TV in the evenings, I will study. I failed in the examination. Therefore I must have watched TV in the evening. **(07 Marks)**

c. Verify the argument is valid.

$\forall x, [p(x) \vee q(x)]$
$\exists x, \neg p(x)$
$\forall x, [\neg q(x) \vee r(x)]$
$\forall x, [s(x) \rightarrow \neg r(x)]$
$\therefore \exists x, \neg s(x)$ **(07 Marks)**

3 a. Define symmetric difference of two sets. Determine the sets A and B, given that $A - B = \{1, 2, 4\}$, $B - A = \{7, 8\}$ and $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$. **(06 Marks)**

b. In a set of 50 students, 15 study mathematics, 8 study Physics, 6 study Chemistry and 3 study all these three subjects. Prove that 27 or more students study none of these subjects. **(07 Marks)**

c. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. If is required to answer seven questions selecting at least two questions from each part. In how many different ways can a student select his seven questions for answering? **(07 Marks)**

4 a. Prove that, for each $n \in Z^+$, $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6} n(n + 1)(2n + 1)$ using mathematical induction. **(06 Marks)**

b. Give a direct proof of the statement "The square of an odd integer is an odd integer". **(07 Marks)**

c. Find the recurrence relation and the initial condition for the sequence 0, 2. 6, 12, 20, 30 ...... Hence find general term. **(07 Marks)**

5 a. Let A = {1, 2, 3, 4} and let R be the relation on A defined by xRy iff x > y.
i) Write down R as a set of ordered pairs
ii) Draw the digraph of R. **(06 Marks)**

b. If A = {1, 2, 3, 4} and R, S are relations on A defined by R = {(1, 2), (1, 3), (2, 4), (4, 4)}, S = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)}. Find RoS, SoS. Draw its digraph and write down their matrices. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. On this set define the relation R by (x, y) ∈R iff x – y is a multiple of 5. Verify that R is an equivalence relation. **(07 Marks)**

6 a. Define sterling number of second kind. Prove that if 30 dictionaries in a library contain a total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages. **(06 Marks)**

b. Define one–one and onto function. Consider the relations on the set A = {1, 2, 3} f = {(1, 3), (2, 3), (3, 1)}, g = {(1, 3), (2, 2), (3, 1)} Which of these are one–one and onto functions? **(07 Marks)**

c. Consider the function $f : R \to R$ defined by $f(x) = 2x + 5$. Let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f. **(07 Marks)**

7 a. Define complete graph, complete bipartite graph and connected graph with one example. **(06 Marks)**

b. Find the chromatic polynomial for the cycle C_4. **(07 Marks)**

c. Find all paths of length 4 and all the cycles in the graph Fig. Q7(c). **(07 Marks)**

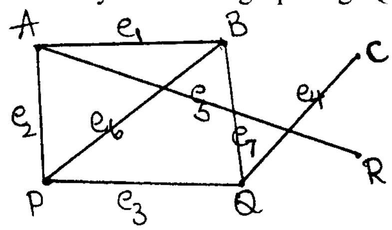

Fig. Q7(c)

8 a. Define a tree. Prove that a tree with 'n' vertices has n – 1 edges. **(06 Marks)**

b. Apply merge–sort to the list –1, 7, 4, 11, 5, –8, 15, –3, –2, 6, 10, 3. **(07 Marks)**

c. Obtain an optimal prefix code for the message "MISSION SUCCESSFUL". Indicate the code **(07 Marks)**

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